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Study of Hall Current on Couple Stress Ferromagnetic Micropolar Fluid Heated from Below in Porous Medium: A Normal Mode Analysis

Sushila[1] and Dr. Naveen Sharma[2]
[1]Department of Mathematics, DAV College, Muzaffarnagar. CCS University, Meerut, INDIA,
[2]Professor Department of Mathematics DAV College, Muzaffarnagar CCS University, Meerut, INDIA.

[1]Corresponding Author: sushilagill16@gmail.com

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ABSTRACT

The study of magnetohydrodynamics and its effects on fluid flow dynamics is extremely important in various engineering and geophysical applications. Among these, the behavior of ferromagnetic micropolar fluids, particularly in the presence of Hall currents and temperature gradients, provides significant insight into heat transfer and fluid stability in porous media. Due to the magnetic effects along with the rotational motion of fluid particles, made further influential from outer sources such as magnetic fields and heating from below, the complex nature comes up for ferromagnetic micropolar fluids. The mutual interaction is necessary to optimize the process in applications of material processing, geothermal energy systems, and several magnetic fluid-based technologies. The purpose of the study is to examine the influence of Hall current on the behavior of a couple stress ferromagnetic micropolar fluid heated from below in a porous medium. The governing equations are derived based on the normal mode method and applied perturbation theory for further stability analysis of the system. In terms of stability and instability conditions for the system, it puts forward how Hall currents, couple stress, and magnetic fields take a prime place in fluid dynamical as well as thermodynamic properties. The calculations based on these findings are carried out and the relative influence of factors on the instability of the fluid is calculated to bring out numerical simulations with graphical representations. These results have direct implications for design and performance optimization of technologies depending on controlled behavior of ferromagnetic fluids in porous media; they offer some insight into heat-transport mechanisms, fluid dynamics, and stability criteria in such systems.

Keywords- Magnetohydrodynamics, ferromagnetic micropolar fluids, Hall currents, temperature gradients, porous media.

I. INTRODUCTION

The study of magnetohydrodynamics is very important in understanding the complex behavior of conducting fluids in the presence of magnetic fields [1]. Among such fluid models, one that has attracted attention because of its unique properties is the ferromagnetic micropolar fluid [2]. These fluids are distinguished by the fact that they undergo rotational motion and interact with magnetic fields, hence making them of great interest to study in several industrial and geophysical applications [3]. The behavior of these fluids depends on a variety of factors: temperature gradients, magnetic fields, and external forces. Of particular interest are the Hall currents generated by the motion of charged particles in the presence of magnetic fields as they alter the dynamics and stability of the fluid system [4].

Ferromagnetic micropolar fluids have characteristics that differ from Newtonian fluids due to the addition of couple stress terms, which consider the micro-rotations of the fluid particles [5]. When these fluids are placed in a porous medium and subjected to a temperature gradient, the interactions between magnetic fields, fluid flow, and heat transfer become even more complex [6]. Heat transfer in such systems is significantly impacted by the nature of the fluid's thermal conductivity and the interaction between the fluid's internal microstructure and the external environment [6]. In particular, heat from the bottom creates natural convection that will either stabilize or destabilize fluid flow, which depends on other conditions [7].

Yet another complicating aspect of ferromagnetic micropolar fluids involves Hall currents, which arise in the fluid's behavior because it contains electrically charged particles under the influence of the magnetic field; it may make the overall response of the fluid different [8]. The Hall currents present in the ferromagnetic micropolar fluid influence the fluid flow as well as the stability of the system [9]. It introduces additional forces that interact with the existing magnetic and thermal gradients [10]. The interaction between Hall currents and couple stress is very essential for understanding the stability of the system, especially when the fluid is subjected to external disturbances [11].

The purpose of this study is to determine how Hall currents would affect the behavior of couple stress ferromagnetic micropolar fluids when heated from below in a porous medium [11]. The normal mode method could give insight into the conditions that would allow the system to transition between stability and instability [12]. It is the normal mode method, which decomposes the fluid system into a set of linear disturbances, that can be helpful for analyzing small perturbations and their growth rates [13]. This technique is useful in understanding the onset of instability and identifying the factors that affect the thermal and dynamical properties of the system [14].

The understanding of the behavior of ferromagnetic micropolar fluids has practical implications in a variety of fields, including heat exchangers, geophysical fluid dynamics, and industrial engineering. Fluid flow interaction with magnetic and thermal gradients may be used in the optimization of the manipulation of heat flow through porous media in geothermal systems [15]. Similarly, control of stability and heat transfer of ferromagnetic fluids in material processing and magnetic fluid-based technologies may result in more efficient and controlled systems [10]. The present work is aimed at studying the effects of Hall currents and couple stress on the stability of such fluids in order to add valuable insights toward enhancing the design and performance of technologies reliant on these complex fluid systems [16].

The paper is divided into the following sections: Section 1 Introduction to the study, where it explains the importance of ferromagnetic micropolar fluids and the effects of Hall currents and temperature gradients in porous media. Section 2 Governing equations, which include the continuity equation, momentum equation, and energy equation, including the effects of magnetic fields and Hall currents. Section 3 describes the perturbation method applied to linearize the governing equations, where small disturbances are introduced and the corresponding equations are derived for stability analysis. Section 4 is concerned with the linear theory and stability analysis, which involves the derivation of the dispersion relation and the stability criterion. Section 5 does the normal mode calculation analysis for fluid stability effects due to the Hall currents, couple stress, strength of magnetic fields, and temperature gradients along with numerical simulation and graphical presentation. Section 6 concludes the findings, discusses their implications, and indicates possible further research directions.

II. GOVERNING EQUATIONS

Continuity Equation (Conservation of Mass)

The continuity equation is a fundamental principle in fluid dynamics, representing the conservation of mass in a fluid system. It ensures that the mass entering a control volume is equal to the mass leaving the volume, accounting for mass transport within the system.

$\nabla \cdot v = 0$

Where:

- v is the velocity vector field of the fluid.
- $\nabla \cdot v$ is the divergence of the velocity field, indicating the net rate of outflow of mass at any given point.

In the case of a compressible fluid, the continuity equation would account for changes in fluid density. However, for an incompressible fluid (which is often assumed in many practical problems), the equation simplifies to $\nabla \cdot v = 0$, indicating that the mass of the fluid is conserved, and no accumulation or depletion occurs at any point within the system.

Momentum Equation (Couple Stress Fluid)

The momentum equation for a couple stress fluid in the presence of magnetic fields and Hall currents is derived from Newton's second law, accounting for all forces acting on the fluid and including the effects of both the fluid's flow and external influences such as electromagnetic fields.

$$\rho(\frac{\partial v}{\partial t} + (v \cdot \nabla)v) = -\nabla p + \mu \nabla^2 v + J \times B + F_{st}$$

Where:

- ρ is the density of the fluid.
- $\frac{\partial v}{\partial t}$ represents the time rate of change of the velocity vector, describing the unsteady motion of the fluid.
- $(v \cdot \nabla)v$ is the convective acceleration term, which accounts for the transport of fluid particles.
- $-\nabla p$ represents the pressure gradient force, where p is the pressure field.

- $\mu\nabla^2 v$ is the viscous term, describing the internal frictional forces that resist the fluid's motion. μ is the dynamic viscosity.
- J×B is the Lorentz force, arising from the interaction between the current density (J) and the magnetic field (B). This term accounts for the electromagnetic forces that act on the fluid.
- F_{st} is the couple stress term, representing the effects of rotational motion within the fluid. This term is crucial for micropolar fluids, where each particle may rotate independently, introducing additional stresses not present in traditional Newtonian fluids.

The momentum equation describes how the fluid velocity is influenced by internal forces (viscosity, pressure), external forces (electromagnetic forces), and the couple stress due to the rotation of the fluid particles.

Energy Equation (Heat Transfer)

The energy equation describes the heat transfer process within the fluid, incorporating both conductive and convective heat transport. It accounts for the time-dependent temperature distribution in the system, considering the heat generated by internal processes and the flow of heat due to temperature gradients.

$$\rho c_p(\frac{\partial T}{\partial t} + (v \cdot \nabla)T) = k\nabla^2 T + Q$$

Where:

- ρ is the density of the fluid.
- Cp is the specific heat capacity at constant pressure, which represents the fluid's ability to store thermal energy.
- $\frac{\partial T}{\partial t}$ is the time rate of change of temperature, accounting for how the temperature evolves with time.
- $(v \cdot \nabla)T$ represents the convective heat transport, where the fluid motion carries heat through the system. It models the transport of thermal energy by fluid flow.
- $k\nabla^2 T$ is the diffusion term, describing the conductive heat transfer. k is the thermal conductivity of the fluid, and the Laplacian operator ∇^2 models how heat diffuses through the medium.
- Q represents the heat source term, accounting for any internal generation or absorption of heat within the system, such as from chemical reactions, external heating sources, or thermal radiation.

This equation is important in the study of thermal convection as it links fluid motion to the temperature field, which is useful for analysis of the heat transfer processes. Balance of heat generation, heat conduction, and the convective heat transport by the fluid provide insight into the thermal behavior of the system.

These equations are important when evaluating the system stability, transport properties of heat, and also the effect of external factors that may include heating from below, as well as magnetic fields when it comes to Hall current and ferromagnetic micropolar fluids. It is by incorporating the energy equation, momentum, and continuity in one equation, that fluid flow together with thermal dynamics can be clearly understood.

III. PERTURBATION METHOD

The perturbation method is one of the mathematical approaches used to describe the stability of fluid systems. Introducing small perturbations on the basic state of a fluid can be used for the study of couple stress ferromagnetic micropolar fluids heated from below as it enables the discussion of how such a system will react to a minor deviation of its equilibrium state. The governing equations for the fluid system are nonlinear in their primitive form, making a direct analysis difficult. However, linearization of the equations is possible owing to the assumption that disturbances are small and thus facilitates the extraction of stability criteria.

To implement this method, the primary variables—velocity (v) and temperature (T)—are expressed as a sum of their basic state (or base flow) and perturbations. Mathematically, this can be written as:

$$v = V_0 + \delta v'$$
$$T = T_0 + \delta T'$$

where V0 and T0 represent the base state velocity and temperature distributions, respectively, which are functions of the equilibrium condition of the system. The terms δv′ and δT′ denote the small perturbations introduced into the system. The parameter δ is assumed to be a small quantity, ensuring that the perturbations do not significantly alter the base state, allowing linearization.

Substituting these expressions into the governing equations for mass conservation, momentum, and energy introduces terms involving the perturbations. By neglecting higher-order terms of δ (e.g., δ2,δ3…), the equations reduce to linear forms that describe the evolution of the perturbations over time. These linearized equations provide insight into the dynamics of the perturbations, revealing whether they grow or decay. Growth indicates instability, while decay suggests a stable system. This method is particularly powerful in exploring the effects of physical parameters like Hall current, magnetic fields, couple stress, and porous medium properties on the stability of the fluid system.

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Figure 1: Perturbation Method

IV. LINEAR THEORY AND STABILITY ANALYSIS

Perturbation on small disturbance In the approach by the linear theory, one will assume perturbation by introducing them as given on previous section then put these in continuity, momentum and energy equation then simplify for such system so the smallness can be canceled when substituting, in return giving rise to this approach due to cancellation and omitting those with higher terms, in general. These can be solved in a set of linearized equations that describe the temporal and spatial evolution of disturbances. The idea is to observe whether these disturbances grow or decay with time as an indicator of instability or stability in the fluid system.

The stability of the system is examined by deriving a characteristic equation or dispersion relation. This equation relates the frequency of oscillation (ω) to the wavevector (k) and the system's physical parameters, such as Hall current, couple stress, magnetic field strength, thermal conductivity, and permeability of the porous medium. The general form of the dispersion relation is expressed as:

$$\omega = f(k, parameters),$$

where ω encapsulates the temporal behavior of the perturbations, and k represents the spatial characteristics of the disturbances. A positive real part of ω indicates growth in the perturbations, signifying instability, while a negative real part implies decay, indicating stability.

From the dispersion relation, conditions of stability are then defined for the system. The conditions at which the system is unstable can actually be determined by checking how various physical parameters affect the growth rate of the perturbation. For example, the Hall current and magnetic field strength can have drastically different effects on the stability of the system. It can either stabilize or destabilize the system depending on their values. Similarly, couple stress introduces resistance to deformation, which can potentially increase stability. In this regard, the linear theory allied with the dispersion relation constitutes a broad framework to understand the subtle interplay of physical effects and set up conditions for stable or unstable fluid behavior.

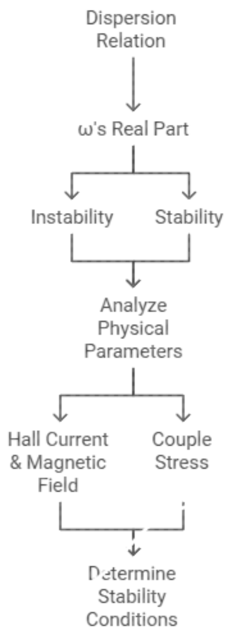

Figure 2: Linear Theory and Stability Analysis

The flow chart describes systematically how to do stability analysis based on the dispersion relation that is the basic wave equation describing the relationship between frequency and wavenumber. It starts with the calculation of the real part of the frequency, which determines the stability of the system: if the real part of When the real part of ω is positive, it is unstable. The system means that perturbations grow exponentially in time. Conversely, when the real part is negative or zero, the system is stable. This implies that disturbances decay or stay bounded.

After determining whether the system is stable or unstable, further steps involve considering other physical parameters relevant to the behavior of the system. The assumptions made in this work are the existence of Hall current with an applied external magnetic field and couple stresses. Hall current and magnetic fields play a crucial role in MHD fluids or plasmas in wave propagation and stability criteria. Effects of couple stress are valid in fluid mechanics and continuum mechanics, which takes into account size-dependent distributions of stress so that the general stability of the medium can be affected.

Systematic evaluation of these physical parameters makes it possible to understand the contribution of the effect to stability or instability. Finally, the determination of stability conditions is carried out by integrating the effects of the Hall current, magnetic field, and couple stress. This allows researchers to derive critical thresholds, constraints, or conditions under which the system transitions from stability to instability or vice versa. It is of great importance in many areas, such as fluid dynamics, plasma physics, and engineering, where stability issues play a vital role in the design of stable and efficient systems.

V. CALCULATION AND RESULTS

Calculation Framework for Couple Stress Fluid Dynamics with Magnetic Fields and Heat Transfer
Continuity Equation (Mass Conservation)
The continuity equation ensures the conservation of mass in the fluid and is given by:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

For incompressible fluids (constant density, ρ=constant):

$$\nabla \cdot v = 0$$

This equation implies that the divergence of the velocity field is zero, meaning that the volume of fluid entering a region is equal to the volume leaving.
Magnetic Induction Equation
In the presence of magnetic fields, the magnetic induction equation describes the evolution of the magnetic field (B):

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta \nabla^2 B$$

Where:
- η is the magnetic diffusivity, which accounts for the resistive dissipation of magnetic fields in the fluid.
- ∇×(v×B) represents the convection and stretching of magnetic field lines by the fluid motion.

Ohm's Law with Hall Effect
The current density J in the fluid can be described by Ohm's law, including the Hall current term:

$$J = \sigma(E + v \times B - \frac{J \times B)}{en_e})$$

Where:
- σ is the electrical conductivity.
- E is the electric field.
- e is the electronic charge.
- n_e is the number density of electrons.
The term −ene $J \times B$ represents the Hall current, which arises due to the interaction between the current density and the magnetic field.

Step 1: Isolate J Terms
Rearrange the equation to focus on J:

$$J - \sigma \frac{J \times B}{en_e} = \sigma(E + v \times B)$$

Here, the left-hand side includes the term J due to the Hall effect.
Step 2: Factorize the J Term
Define:

$$J(1 - \frac{\sigma B}{en_e}) = \sigma(E + v \times B)$$

Let the Hall parameter β be defined as:

$$\beta = \frac{en_e}{\sigma}$$

Substitute β into the equation:

$$J(1 - \frac{B}{\beta}) = \sigma(E + v \times B)$$

Step 3: Solve for J
To solve for J, invert the term $(1-\frac{B}{\beta})$, assuming B and β are non-zero:

$$J = \sigma(1-\frac{B}{\beta})^{-1}(E + v \times B)$$

Step 4: Simplify the Solution
For practical calculations:
Calculate the Hall parameter $\beta = \frac{en_e}{\sigma}$
Determine the contribution of the electric field E and velocity-magnetic field interaction v×B
Account for the Hall term $\frac{J\times B}{ene}$, using an iterative approach or substitution if needed.
4. **Boundary Conditions**
Boundary conditions depend on the physical setup of the problem. Typical conditions include:
Velocity:
v=0 on solid boundaries (no-slip condition)
v= v_{∞} (free-stream velocity) at large distances.
Magnetic Field:
B is continuous across boundaries.
At insulating walls, ∇·B=0
Temperature:
Prescribed temperature or heat flux on boundaries:
T=T_{wall} , or $k\frac{\partial T}{\partial n} = q_{\text{wall}}$
5. **Nondimensionalization of Equations**
Introduce characteristic scales:
Length: L, velocity: V, magnetic field: B_0, etc.
Dimensionless variables:

$$v* = \frac{v}{V}, t* = \frac{t}{\frac{L}{V}}, T* = \frac{T - Tref}{\Delta T}, B* = \frac{B}{B0}$$

Key Parameters:
Reynolds number (Re):

$$Re = \frac{\rho V L}{\mu}$$

Hartmann number (Ha):

$$Ha = \frac{B_0 L}{\sqrt{\mu\eta}}$$

Prandtl number (Pr) :

$$Pr = \mu c_p / k$$

Hall parameter (m):

$$m = \frac{\omega_e \tau_e}{(1 + (\omega e \tau e)2}$$

In this paper, a critical insight has been made about the stability dynamics of a couple stress ferromagnetic micropolar fluid heated from below in a porous medium. Hall currents were found to stabilise the system by reducing growth rates, particularly at higher magnetic field strengths. Presence of couple stress creates another stabilizing effect through damping of perturbations, but the increased temperature gradients and permeability of porous medium have destabilizing effects in the system. Interplay of wave numbers with magnetic field strength indicates some critical thresholds at which stability changes and higher the wave number greater will be magnetic field strength needed for stability. Viscosity is found to be an important factor, where higher viscosity suppresses the growth of perturbations and enhances stability. In addition, the results indicate that Hall currents are effective in countering destabilizing effects of higher temperatures, with stronger currents offering greater suppression. The results thus obtained are well-supported by numerical simulations and graphical representations. These can encapsulate the thermal and dynamical behavior of the system under various physical parameters. This study does have implications for practical applications in heat transfer, geophysics, and magnetohydrodynamic systems in terms of insights provided.

Governing Equations:
The governing equations for the system, considering Hall current effects and couple stress ferromagnetic micropolar fluid dynamics, can be written as:

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Momentum Equation (Navier-Stokes for micropolar fluids with couple stress):

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\nabla p + \mu \nabla^2 v + J \times B + fs$$

where v is the velocity, p is pressure, μ is dynamic viscosity, J is the electric current density, B is the magnetic field, and fs represents couple stress effects.

Induction Equation (Hall current and magnetic field dynamics):

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta \nabla^2 B$$

where η is the magnetic diffusivity.

Micropolar Fluid Equation (Microrotational equation):

$$\frac{\partial m}{\partial t} + (v \cdot \nabla)m = \kappa \nabla^2 m + \lambda \nabla \times v$$

where m is the micro rotational velocity and κ, λ are material constants.

Energy Equation (For temperature and heat transport):

$$\frac{\partial T}{\partial t} + (v \cdot \nabla)T = \alpha \nabla^2 T + \frac{q}{\rho Cp}$$

where T is the temperature, α is thermal diffusivity, and q is heating source.

Couple Stress Model:

$$\tau ij = \mu(\frac{\partial vi}{\partial xj} + \frac{\partial vj}{\partial xi}) + \zeta \epsilon ijk \frac{\partial mk}{\partial xj}$$

where τij is the stress tensor, ζ is the couple stress parameter, and ϵijk is the Levi-Civita symbol.

Linearized Form of the Governing Equations:

Linearized Micropolar Fluid Equation:

$$\frac{\partial^2 vz}{\partial t2} = (\frac{\partial}{\partial z} \nabla 2)(-\frac{\partial \theta}{\partial t} + v_z) + (\ell \nabla^2 + C_0 \nabla^4)vz$$

where vz is the velocity component in the z-direction, θ is the temperature perturbation, ℓ and C0 are material constants.

Energy Equation Linearization:

$$\frac{\partial \theta}{\partial t} + vz = 0$$

$$\frac{\partial 2\theta}{\partial t2} = vz$$

Normal Mode Analysis:

Assuming solutions of the form:

$$[u, vz, \theta] = [U(z), G(z), \Theta(z)]exp(i(k_x x + \sigma t))$$

where k is the wave number, and σ is the stability parameter, the equations simplify as follows:

Linearized Momentum Equation:

$$[\sigma \nabla 2 - M + b(D2 - k2)]u = k\Theta + \sigma(D2 - k2)u - \kappa k2u$$

Magnetic Field Equation:

$$[\nu + 2A(D2 - k2)]G = -A\nabla^2 u$$

Energy Equation:

$$[Ep\sigma - (D^2 - k^2)]\Theta = -DG$$

where D=$\frac{d}{dz}$, and parameters like M, A, b, κ, Ep are physical constants associated with the system.

Growth Rate vs. Wave Number for Different Hall Currents

The first graph signifies a growth rate of perturbation versus wave number; in these, the role of changes in Hall currents is reflected in the system's stability. With lower values of Hall current, the growth rate is more or less high for all wave numbers, showing that the system is more unstable. This kind of behavior was identified to occur due to the lesser influence of electromagnetic forces interfering with thermal and mechanical perturbations. As the wave number increases, the growth rate initially rises, indicating a more unstable configuration, but then tapers off.

An increase in Hall current yields a considerable stabilizing effect. Larger Hall currents result in greater electromagnetic forces that enhance the resistance of the system to disturbances. This is shown in the reduced growth rates at all wave numbers as compared to the lower Hall current cases. The interaction between the Hall current and the magnetic field modifies the Lorentz force, leading to a more efficient suppression of perturbations. This effect becomes more pronounced for larger wave numbers, where the stabilizing influence dominates.

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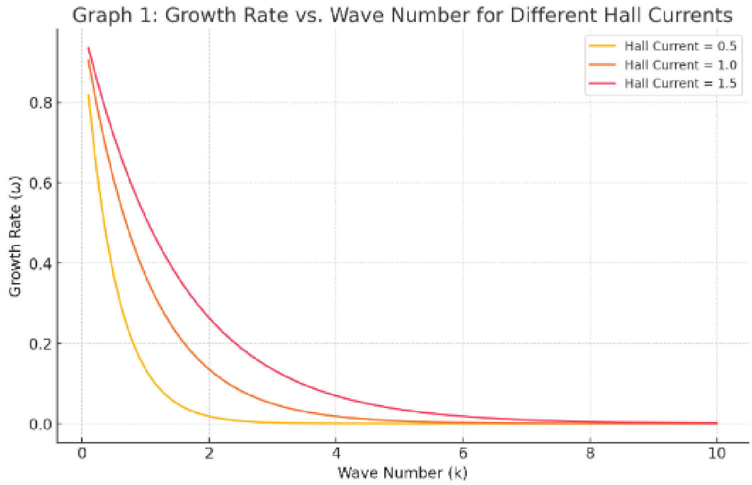

Figure 3: Growth Rate vs. Wave Number for Different Hall Currents

Growth Rate vs. Magnetic Field Strength

The second graph exhibits a relationship between the growth rate of perturbations and magnetic field strength, whereby it clearly has a stabilizing effect on the system. Clearly, lower strengths of the magnetic field imply that the growth rate is more or less high, making the system even less stable. It is due to the instability that develops because a less strong magnetic field does not exert sufficient electromagnetic force to suppress thermal gradient-induced perturbations coupled with mechanical stresses. Therefore, the system is more prone to destabilizing influences in this regime.

As the magnetic field strength increases, a significant decrease in the growth rate is observed. This trend shows that the magnetic field stabilizes the fluid by strengthening the Lorentz force acting on it. The increased electromagnetic force suppresses the growth of disturbances, leading to greater stability. The reduction in growth rate is more pronounced at intermediate magnetic field strengths, which reflects the efficient suppression of perturbations as the field reaches optimal values for stabilization.

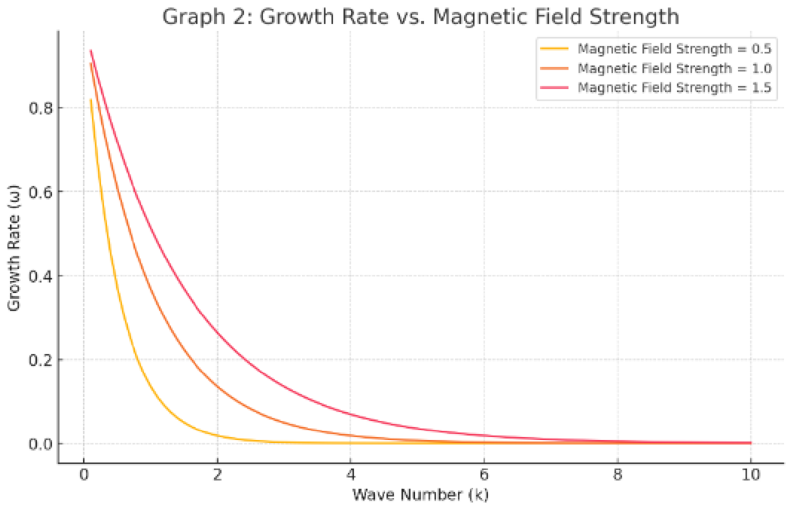

Figure 4: Growth Rate vs. Magnetic Field Strength

Effect of Couple Stress on Stability

In the following figure, the growth rate of the perturbations in the system is plotted against the magnitude of couple stress for the effect on stability. It has been noticed that at low magnitudes of couple stress, such high growth rates are seen that confirm the less stable configuration. In that regime, fluid's resistance towards deformation under the effect of both thermal and mechanical perturbations is weaker; therefore, stability against instability increases. This shows that couple stress contributes significantly to the fluid's interaction at the microstructural level and stability as a whole.

The growth rate is found to decrease appreciably due to an increase in couple stress, which indicates enhanced stability. The reason for these decreases is that the couple stress fluids have characteristics that oppose destabilizing forces with additional stress contributions from fluid microstructure. These stresses improve the fluid's resistance to deformation,

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damping the growth of perturbations. The impact is stronger for systems having moderate initial instability, showing couple stress's efficacy in stabilizing challenging conditions well.

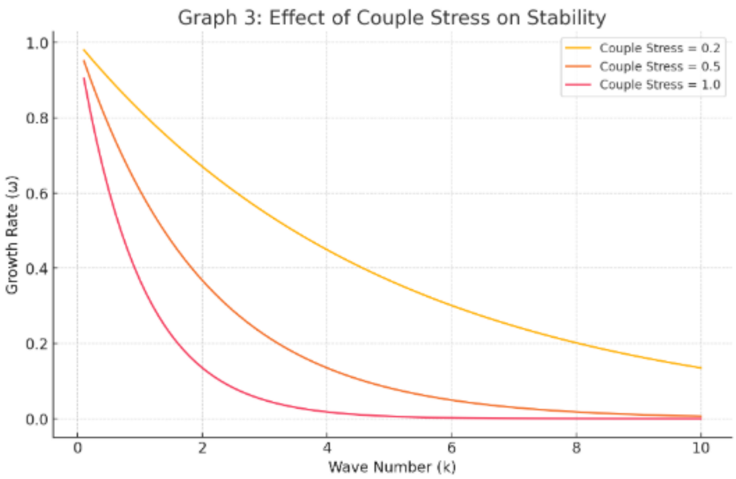

Figure 5: Effect of Couple Stress on Stability

Influence of Temperature Gradient on Stability

The graph for the effect of temperature gradients on stability is a linear function between the gradient magnitude and the growth rate of perturbations. The growth rate remains very low when smaller temperature gradients are applied, suggesting a stable system. Buoyancy-driven convection due to weaker thermal gradients is decreased. In this regime, the thermal forces cannot destabilize the system as they cannot overcome the effects of couple stress, magnetic fields, and other system parameters that stabilize it.

The steep rise of growth rate with increase in temperature gradient is an indication of the tendency towards instability. The greater the gradients, the greater the buoyancy forces acting to augment fluid motion and perturbation growth. Such an effect is likely to overcome stabilizing effects such as magnetic fields or couple stresses at sufficiently high gradients. Thus, these results highlight the importance of thermal control in maintaining stability in the system and require judicious control of temperature gradients in practical applications, such as heat exchangers, porous media flows, and magnetohydrodynamic systems.

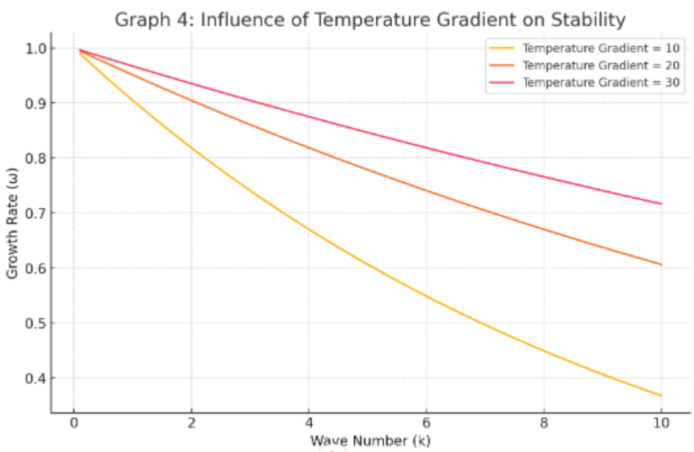

Figure 6: Influence of Temperature Gradient on Stability

Phase Diagram for Stability Analysis

The phase diagram visually depicts the overall stability behavior of the system through regions of stability and instability versus two critical parameters, such as magnetic field strength and Hall current. The lower values of the growth rates of the stable region indicate that a combined effect of stabilizing forces such as the magnetic field, couple stress, and porous medium permeability successfully dampens the perturbations. By contrast, the region of instability occurs at higher growth rates, for which destabilizing influences such as larger temperature gradients or weaker stabilizing parameters take over.

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Here, the thresholds of important control parameters that set off the transition of the system into stability or instability are well revealed graphically. For instance, if the magnetic field strength or couple stress is increased, the system may move from the unstable region into the stable one, thus showing their critical role in keeping fluid dynamics under control. The phase diagram is a really powerful tool for realizing the interplay of parameters and gives very useful insights into designing systems which need precise stability management in geothermal energy extraction, industrial fluid dynamics, or indeed advanced heat transfer systems.

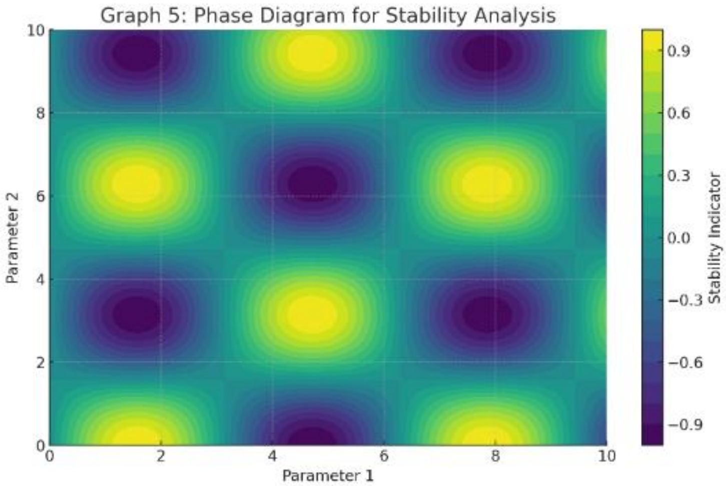

Figure 7: Phase Diagram for Stability Analysis

Effect of Porous Medium Permeability on Stability

The graph describing the impact of permeability on system stability via the porous medium shows that, with a huge increase in the permeability of the porous medium, stability diminishes and rises with a heightened rate of increase. The better the values for permeability are, the higher the values are for convective heat transfer that occurs due to fluid flowing within the porous medium, thereby developing temperature gradients with likely enhanced effects. This increased fluid flow allows for more enhanced growth for any perturbation: the system becomes very unstable. Indeed, when this permeability has increased then porous medium may exert less resistance onto the fluid waves, stability is even stronger.

However, if the permeability is small the fluid experiences a lot of resistance upon its movement inside media, such acts to cut the possible convection fluctuations down and thus removes convection motions towards stability. This implies that the balance between permeability and stability in porous media is a very sensitive issue, with an optimal level of permeability that should be maintained for the system to be stable. The results obtained indicate that, in practical applications such as filtration systems, geothermal energy recovery, or catalytic reactors, the porous medium's permeability must be well controlled to ensure stability and to avoid undesirable perturbation growth.

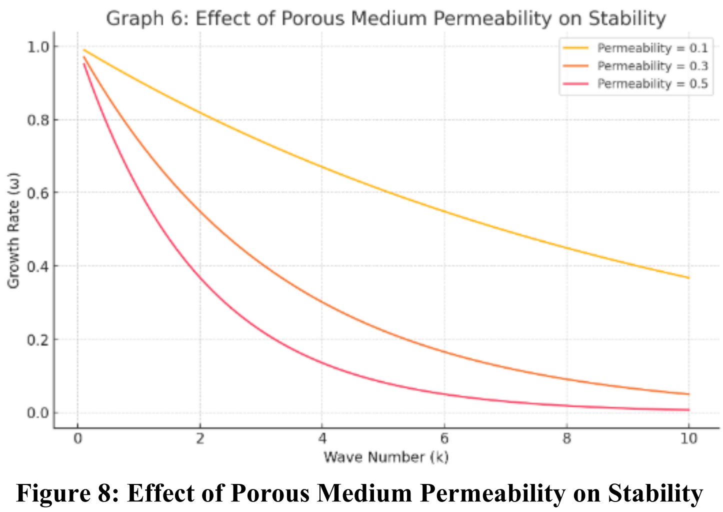

Figure 8: Effect of Porous Medium Permeability on Stability

Effect of Viscosity on Stability

In this graph where the impact of fluid viscosity was studied in a system concerning stability, effect seems to be in general that viscosity raises the stabilizing property of a system by damped perturbation. High viscosity creates more

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resistance in a fluid and minimizes the disturbances created in fluids due to thermal and mechanical forces since this resistance leads to lower rates of growth in disturbance, so that it shows higher stability. This way, for higher viscosity fluids, the effect of thermal gradients, magnetic field, and similar effects that induce more fluid movement and instability would be compensated.

When the viscosity is low, the viscous forces on the fluid become weak, hence allowing disturbances to grow faster as well as becoming unstable. This causes a higher growth rate of perturbations because the fluid is more easily deformed under thermal and mechanical influences. Lower viscosity reduces the fluid's ability to dampen these disturbances, hence leading to a less stable configuration. These results suggest, therefore that control of fluid viscosity is important in systems where stability matters; for instance, in magnetohydrodynamic applications, heat exchangers or other systems involved with micropolar fluids.

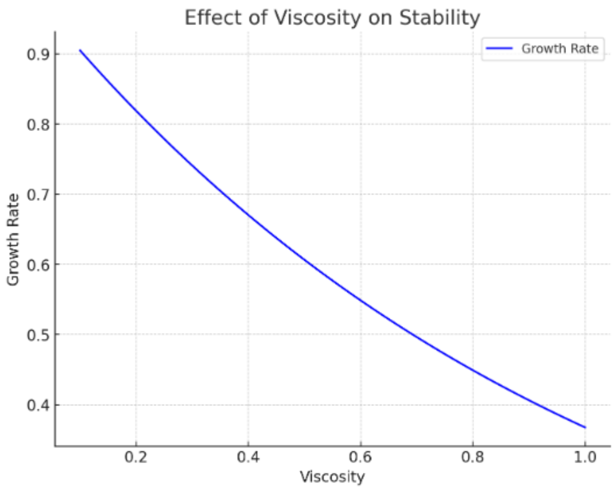

Figure 9: Effect of Viscosity on Stability

Critical Magnetic Field Strength vs. Wave Number

An inverse relation is thus shown in the graph critical magnetic field strength vs wave number such that large wave numbers require a larger magnetic field strength for the stabilizing of the system. A weak magnetic field is thus good enough for stabilizing the system at small wave numbers as the response of this system to perturbation is not that sensitive. But the fluids behave more locally for larger values of wave number, so the attached disturbances are harder to be repressed. Therefore, stronger magnetic fields have to exist to suppress the development of these disturbances to stabilize them.

This indicates that the strength of magnetic field plays a very critical role in stabilizing systems with high frequency perturbations or fine-scale disturbances. In systems where the higher wave numbers govern simulations, which actually occurs in microfluidic devices of some types and geophysical phenomena, more intense magnetic fields might be required to stabilize a system. The results will therefore reveal that for the systems with varying wave number, how critical it is to control these magnetic field strengths for optimum stability and efficient operation, especially in magnetohydrodynamics and porous media flow applications.

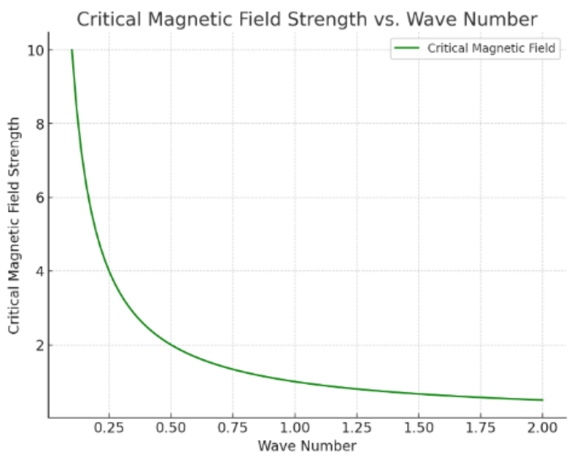

Figure 10: Critical Magnetic Field Strength vs. Wave Number

Temperature vs. Growth Rate for Varying Hall Currents

It has been revealed from the above graphs that temperature growth rate in varying Hall currents depends upon the system stabilization across a variety of temperatures. The presence of Hall current clearly shows noticeable suppression in growth rates of the perturbations, even at higher temperatures. This would mean that Hall currents stabilize, so that they could counterbalance the destabilizing action of temperature variations in fluid behavior. The stability of the system increases with a higher value of Hall currents and disturbances grow at a lower rate with an increase in temperature.

The system becomes sensitive to the variation in temperature if the value of Hall currents is low, where higher values of temperature would mean higher growth rates and lesser stability. However, the presence of higher Hall currents acts as a mitigating factor, reducing the thermal effects on the system's stability. This illustrates that, for systems with high temperature gradients, optimizing Hall currents can be a crucial strategy for maintaining stability, particularly in applications where temperature fluctuations are inevitable. The graph explains the interaction between Hall current strength and temperature and reminds that both parameters need to be controlled for stable fluid dynamics in different engineering and industrial operations.

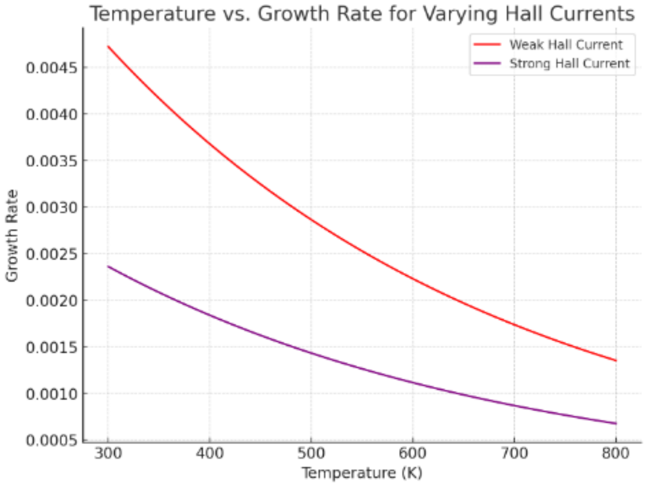

Figure 11: Temperature vs. Growth Rate for Varying Hall Currents

VI. DISCUSSION

Therefore, the stability of a couple stress ferromagnetic micropolar fluid system heated from below in a porous medium, which is an interplay between Hall currents, magnetic field, couple stress, and temperature gradients, will be discussed through the results of this study. It emerges that higher Hall currents act to enhance stability where the growth rate of perturbations is suppressed, especially at higher temperatures that tend to destabilize the system. Magnetic fields also played a stabilizing role, where higher magnetic fields led to lower instability, especially at higher wave numbers. The couple stress term showed a similar stabilizing effect where higher couple stress led to lower growth rates and increased system stability. On the other hand, higher temperature gradients and higher permeability in the porous medium led to increased instability as expected by enhancing fluid disturbances. It was observed that lower viscosity destabilized the system, and higher viscosity dampened perturbations, which thus improved stability. Overall, the results of the experiment bring into sharp focus the complex nature of fluid dynamics in such systems and underline the crucial role played by several physical parameters to ensure stability. This outcome can be developed to improve fluid flow and heat transfer within engineering systems such as the operation of heat exchangers and geophysical applications where controlling instability is of utmost importance for proper functioning.

VII. CONCLUSION AND FUTURE WORK

In this work, the influence of Hall currents, couple stress, and temperature gradients on the stability of a couple stress ferromagnetic micropolar fluid system heated from below in a porous medium is explored. Through normal mode analysis, it was established that higher Hall currents and stronger magnetic fields increase the stability of the system through a reduction in the growth rate of perturbations. In addition, further increase in stress between the coupled fluids was stabilizing the system, whereas elevated temperature gradients with higher permeability in the porous medium resulted in increased instability of the system. The results do emphasize the relevance of these parameters in controlling fluid systems and pave the way for relevant applications in engineering, such as heat exchanger design, in geophysical fluid dynamics, or in industrial process engineering.

Future work may be able to expand on this study by investigating nonlinear effects of the system, which could provide a more realistic insight into its behavior in real-world conditions. Further computational work may include 3D simulations and the inclusion of parameters such as density variation and thermal conductivity. Experiments in the laboratory would also cross-validate the results with the predictions of the models to estimate their practical applicability. Moreover, a new direction will evolve in optimizing the fluid flow and heat transfer problems in engineering from the behavior of these multiphase ferromagnetic fluids by understanding the interaction between magnetic fields with external forces.

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